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Axiomatization of a Preference for Most Probable Winner

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Abstract:

In binary choice between discrete outcome lotteries, an individual may prefer lottery L_1 to lottery L_2 when the probability that L_1 delivers a better outcome than L_2 is higher than the probability that L_2 delivers a better outcome than L_1. Such a preference can be rationalized by three standard axioms (solvability, convexity and symmetry) and one less standard axiom (a fanning-in). A preference for the most probable winner can be represented by a skew-symmetric bilinear utility function. Such a utility function has the structure of a regret theory when lottery outcomes are perceived as ordinal and the assumption of regret aversion is replaced with a preference for a win. The empirical evidence supporting the proposed system of axioms is discussed.

JEL Classification codes: C91, D81

Key words: expected utility theory, axiomatization, betweenness, fanning-in, skew-symmetric bilinear utility, regret theory

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1. Introduction

An individual has menu-dependent preferences when his preference between two choice options depends on the availability of additional options (content of the choice set). The literature often describes such preferences as "context-dependent" (*e.g.* Stewart *et al,* 2003, Tversky and Simonson, 1993). The context of a choice situation is a very general concept, however, that can be also used to describe aspects other than the content of a choice set. A more suitable term to describe a very specific phenomenon—the dependence of individual preferences on the menu of a choice set—is menu-dependence.

In choice under risk (Knight, 1921), a special type of menu-dependent preference is a preference for a lottery that is most probable to outperform all other feasible lotteries. The literature refers to such a preference as "a preference for probabilistically prevailing lottery" (*e.g.* Bar-Hillel and Margalit, 1988) or "the criterion of the maximum likelihood to be the greatest" (*e.g.* Blyth, 1972). Recent experimental evidence suggests that a preference for most probable winner prevails in binary choice between lottery frequencies of equal expected value (Blavatskyy, 2003) and in small feedback-based problems (*e.g.* Barron and Erev, 2003, Blavatskyy, 2003a). In this paper, I build a system of axioms rationalizing a preference for most probable winner in binary choice.

Given the probability distributions of any two independent lotteries it is always possible to calculate directly the (relative) probability of each lottery to outperform the other. Such calculation requires little cognitive effort when the state space (a joint distribution of lotteries) is available. Blavatskyy (2003) provides experimental evidence that a preference for most probable winner emerges when an individual follows a simple majority rule—to pick up a lottery that gives a better outcome in the majority of (equally probable) states of the world. In such a cognitively undemanding environment it is plausible to assume that an individual follows a simple behavioral rule—he calculates the relative probabilities of each lottery to win over the other lotteries and then maximizes among those probabilities. This behavioral rule (the heuristic of relative probability comparisons) resembles one-reason fast and frugal decision making (*e.g.* Gigerenzer and Goldstein, 1996). Like all heuristics, it ignores some of the available information by treating lottery outcomes as ordinal. Additionally, like all heuristics, this behavioral rule applies only to a bounded subset of decision problems, *e.g.* when lotteries have equal or similar expected values.

In cognitively demanding environments, a straightforward calculation of relative probabilities of a lottery to win over others, however, demands more cognitive effort. Examples include situations when probability information is presented visually (*e.g.* Tversky, 1969) or not presented at all (*e.g.* Barron and Erev, 2003), when lotteries have many outcomes or an individual faces a choice among many lotteries. Nevertheless, assuming an individual preference for most probable winner it is possible to explain observed decision making in such environments, as demonstrated in Blavatskyy (2003a) in his alternative explanation of the data in Barron and Erev (2003).

Since individuals are likely to use only simple rules of thumb (*e.g.* Gigerenzer *et al.*, 1999), a descriptive fit of a preference for most probable winner in cognitively demanding decision environments can be explained only through a general theory of preference. Unlike a heuristic approach that describes a plausible psychological process underlying observed decision making (*e.g.* Newell and Shanks, 2003), a theory of preference states that an individual has an underlying preference for most probable winner. The purpose of this paper is to explore the theoretical properties of an individual's preference for most probable winner, and how it is related to various non-expected utility theories (*e.g.* Starmer, 2000). Specifically, the paper explores what normative axioms are necessary and sufficient for rationalizing such preference, and how those axioms accord with the experimental evidence.

The proposed axiomatization provides theoretical insights into an individual's preference for most probable winner and highlights some surprising connections to other decision theories. It also provides "thought experiment" evidence for a descriptive validity of the theory (*e.g.* Friedman and Savage, 1952, Machina, 1982). However, "thought experiment" evidence can be drastically different from actual decision making (*e.g.* Tversky, 1969). Therefore, the paper also focuses on the experimental evidence supposedly documenting the systematic violation of the proposed axioms.

The remainder of this paper is structured as follows. Section 2 introduces the system of axioms and derives a utility function representation and family of indifference curves. The descriptive validity of the proposed axioms is discussed in section 3. Section 4 concludes.

2. The system of axioms

2.1. Basic definitions

An option *A* is strictly preferred to option *B*, or $A \succ B$, if an individual chooses *A* and is not willing to choose *B* from the choice set $\{A, B\}$. An individual is indifferent between choice options *A* and *B*, or $A \sim B$, if the choice of *A* and the choice of *B* are equally possible from the choice set $\{A, B\}$.

This paper deals with individuals' binary choices between discrete lotteries. The set of lottery outcomes $\mathrm{X} = \{x_1,...x_n\}$ is finite and ordered in such a way that $x_1 \prec ... \prec x_n$. Outcomes are not necessarily monetary (measured in reals). They are only required to be strictly ordered in terms of subjective preference. A lottery $L(p_1,..., p_n)$ is defined as a mapping $L : \mathrm{X} \mapsto [0,1]^n$, where $p_i \in [0,1]$ is the probability of occurrence of outcome x_i, $i \in [1, n]$ and $\sum_{i=1}^{n} p_i = 1$.

In a joint independent distribution of any two lotteries $L_1(p_1,..., p_n)$ and $L_2(q_1,..., q_n)$ only three events are possible: L_1 delivers a better outcome than L_2 (state $L_1 > L_2$), L_2 delivers a better outcome than L_1 (state $L_2 > L_1$) and lotteries L_1, L_2 deliver the same outcome (state $L_1 = L_2$). An individual has a preference for most probable winner when equation (1) holds for any two lotteries L_1 and L_2. This decision rule is rationalized below.

$$L_1 \succ L_2 \Leftrightarrow prob(L_1 > L_2) > prob(L_2 > L_1) \Leftrightarrow \sum_{i=1}^{n-1} q_i \left(1 - \sum_{j=1}^{i} p_j\right) > \sum_{i=1}^{n-1} p_i \left(1 - \sum_{j=1}^{i} q_j\right) \quad (1)$$

In the remainder of this paper each pair of lotteries is assumed to be statistically independent. This assumption is not restricting the normative or descriptive applications of the model. A preference for most probable winner is easily extendable on the domain of acts (Savage, 1954) In a binary choice between two acts an individual recodes the outcome of each act as a relative gain (*e.g.* +1) or a relative loss (*e.g.* -1) and then chooses the act that yields a relative gain with the highest probability. Such rule of thumb is intuitively plausible and cognitively undemanding. Therefore, there is no apparent reason for axiomatizing such preference when a joint distribution of lotteries (state space) is given. On the contrary, a preference for most probable winner over independent lotteries is not immediately appealing. This is the main reason why this axiomatization is restricted to independent lotteries.

2.2. Standard axioms

This section presents a set of axioms that were already used in other axiomatizations of decision theories (*e.g.* Fishburn 1982, 1988). Notably, this set of standard axioms does not contain the transitivity axiom. Indeed, a preference for most probable winner can be intransitive (*e.g.* Blyth, 1972). Intuitively, the structure of such preference is foremost based on the relative (binary) comparisons rather than on a separate (menu-independent) evaluation of lotteries. For example, lottery L_1 that yields €4 Euro with probability $\Psi = \left(\sqrt{5} - 1\right)/2 \approx 0.618$ and €1 otherwise is more probable to deliver a higher outcome than €3. Lottery L_2 that yields €2 with probability Ψ and €5 otherwise is more probable to deliver a lower outcome than €3. However, L_2 delivers a higher outcome than L_1 with probability Ψ. Hence, a preference for most probable winner does not necessarily impose a transitive order on lotteries.

Axiom 1 (Solvability) For any three lotteries L_1, L_2, L_3 such that $L_1 \succ L_2$ and $L_2 \succ L_3$ there is a number $\alpha \in (0,1)$ such that $\alpha L_1 + (1-\alpha)L_3 \sim L_2$

Axiom 2 (Convexity) For any three lotteries L_1, L_2, L_3 and for any number $\alpha \in (0,1)$:

a) if $L_1 \succ L_2$ and $L_1 \geq L_3$ then $L_1 \succ \alpha L_2 + (1-\alpha)L_3$,

b) if $L_1 \sim L_2$ and $L_1 \sim L_3$ then $L_1 \sim \alpha L_2 + (1-\alpha)L_3$,

c) if $L_1 \succ L_2$ and $L_3 \geq L_2$ then $\alpha L_1 + (1-\alpha)L_3 \succ L_2$.

Axiom 3 (Symmetry) For any three lotteries L_1, L_2, L_3 such that $L_1 \succ L_2$, $L_2 \succ L_3$, $L_1 \succ L_3$, $L_2 \sim 0.5L_1 + 0.5L_3$ and for any number $\alpha \in (0,1)$:

$\alpha L_1 + (1-\alpha)L_3 \sim 0.5L_1 + 0.5L_2$ if and only if $\alpha L_3 + (1-\alpha)L_1 \sim 0.5L_3 + 0.5L_2$.

Fishburn (1982, 1988) proved the following theorem.

Theorem 1 Axioms 1-3 hold if and only if there is a skew-symmetric function $\psi : \mathrm{X} \times \mathrm{X} \to \mathrm{R}$ (unique up to a multiplication by a positive constant) such that for any two lotteries $L_1(p_1,...,p_n)$ and $L_2(q_1,...,q_n)$: $L_1 \succ L_2$ if and only if $\sum_{i=1}^{n}\sum_{j=1}^{n} p_i q_j \psi(x_i, x_j) > 0$

2.3. A fanning-in axiom

A fanning-in axiom assumes a particular type of diminishing sensitivity to probability. Specifically, when probability mass is largely shifted to the best or the worst outcome, tiny probabilities attached to the intermediate outcomes become progressively unimportant for decision. This axiom has not been used in other axiomatizations in the literature.

Axiom 4 (A fanning-in) For any lottery L_1 that delivers x_i with probability p_i and x_n otherwise, and for any lottery L_2 that delivers $x_j > x_i$ with probability p_j and x_n otherwise, such that $L_1 \sim L_2$: $\lim_{p_i \to 0} p_j / p_i = 1$. For any lottery L_3 that delivers x_k with probability p_k and x_1 otherwise, and for any lottery L_4 that delivers $x_l > x_k$ with probability p_l and x_1 otherwise, such that $L_3 \sim L_4$: $\lim_{p_l \to 0} p_k / p_l = 1$.

To understand the logic behind axiom 4, consider first the situation when $p_i \to 0$. First of all, notice that $p_j > p_i$ because $x_i \prec x_j \prec x_n$. If $p_i \to 0$, lottery L_1 approaches to the lottery $\overline{L}(0,...,0,1)$, which gives the best possible outcome x_n for sure. Since there could be no other lottery $\widetilde{L}_2$ such that $\widetilde{L}_2 \sim \overline{L}$ it must be the case that $\lim_{p_i \to 0} p_j = 0$. When two lotteries L_1, L_2 approach to the lottery $\overline{L}$ the absolute differences in tiny probabilities attached to the not-best outcome disappear. Axiom 4 additionally requires that the relative differences in probabilities attached to the not-best outcome also disappear as L_1 and L_2 become increasingly similar to $\overline{L}$ *i.e.* $\lim_{p_i \to 0} p_j / p_i = 1$.

Expected utility theory violates axiom 4 because it implies that $\lim_{p_i \to 0} p_j / p_i > 1$. If all outcomes are either gains or losses, cumulative prospect theory satisfies axiom 4 only in a special case when $\lim_{p \to 1} \frac{\partial \pi^{-1}}{\partial p} \frac{\partial \pi}{\partial p} = \frac{u(x_n) - u(x_1)}{u(x_n) - u(x_j)}$, where $\pi(p)$ is the probability weighting function and $u(.)$ is value function (*e.g.* Tversky and Kahneman, 1992). When $\lim_{p_i \to 0} p_j / p_i = 1$ then an individual's indifference curves plotted in the probability triangle[1] (*e.g.* Machina, 1982) are not parallel but fanning-in, which explains the name of the axiom. The assumption $\lim_{p_i \to 0} p_j / p_i = 1$ also implies that an individual becomes infinitely risk seeking when probability mass is largely shifted to the best outcome.

The second part of axiom 4 assumes that the above logical argument applies as well to the situation when lotteries $L_3(1 - p_k, 0,...,0, p_k, 0,...,0)$ and $L_4(1 - p_l, 0,...,0, p_l, 0,...,0)$, $L_3 \sim L_4$, approach to the lottery $\underline{L}(1,0,...,0)$, which gives the worst possible outcome x_1 for

[1] Note that lotteries L_1, L_2, although defined as probability distributions over *n* outcomes, have non-zero probabilities attached only to three outcomes x_i, x_j, x_n. Therefore, lotteries L_1, L_2 can be plotted in the probability triangle based only on outcomes x_i, x_j, x_n.

sure. The only difference is that an individual becomes infinitely risk averse when probability mass is largely shifted to the worst outcome. The implication of axiom 4 that an individual becomes risk seeking (averse) when probability mass is largely shifted to the best (worst) outcome is the counterpart of Machina's intuition for universal fanning out (Machina, 1987 pp. 129-130). As shown in the proof of theorem 2 below the intuitive role of axiom 4 is "to erase" the cardinal difference between lottery outcomes. An individual who maximizes the probability of a relative gain ignores the information about the size of this gain i.e. he or she treats lottery outcomes in an ordinal way.

Theorem 2 Axioms 1-4 hold if and only if equation (1) holds for any two lotteries L_1 and L_2.

■ Proof is presented in the appendix. ■

Theorem 2 implies that an individual's preference for most probable winner is a special case of the skew-symmetric bilinear utility theory (*e.g.* Fishburn, 1982, 1988). The addition of a fanning-in axiom restricts a general skew-symmetric bilinear functional derived by Fishburn so that only the ordinal difference in lottery outcomes is taken into account. When lotteries are distributed independently, skew-symmetric bilinear utility theory coincides with regret theory (*e.g.* Loomes and Sugden, 1982, 1987). When the anticipated net advantage function of regret theory is ordinal in outcomes (*e.g.* equation (5) in the appendix), the decision rule of regret theory reduces to a preference for most probable winner. However, such "ordinal" function $\psi(x_i, x_j)$ always violates a key assumption of regret theory, regret aversion, which requires $\forall x > y > z \Rightarrow \psi(x,z) > \psi(x,y) + \psi(y,z)$ (*e.g.* Loomes *et al.*, 1992). In terms of regret theory the "ordinal" function $\psi(x_i, x_j)$ of a preference for most probable winner always reflects regret seeking: $\forall x > y > z \Rightarrow \psi(x,z) < \psi(x,y) + \psi(y,z)$.

Figure 1 plots the map of the indifference curves representing a preference for most probable winner inside the probability triangle (Machina, 1982). The same map of indifference curves is implied by the weighted utility theory when the weight of the medium outcome is greater than unity (*e.g.* Chew and Waller, 1986). Notice that this indifference map is independent of individual-specific parameters (functions) and cardinal measures of lottery outcomes *i.e.* the map is invariant for all triples of lottery outcomes such that $x_1 \prec x_2 \prec x_3$. The family of indifference curves implied by axioms 1-4 consists of straight lines with different slopes reflecting a changing individual attitude towards risk Specifically, a universal fanning-in, as in figure 1, shows that an individual becomes more risk seeking (averse) when probability mass is shifted to the best (worst) outcome, which Chew and Waller (1986) call "the heavy hypothesis".

Figure 1 demonstrates that an individual is risk neutral along the 45° line on figure 1 *i.e.* he or she is exactly indifferent between a medium outcome for sure and a 50%-50% chance of the best and the worst outcome. This is a direct consequence of the symmetry axiom. The symmetry axiom probably gains the most of its intuitive appeal when lottery outcomes are "similar" to each other. Hence, a preference for most probable winner is especially appealing when lotteries have equal or similar expected values.

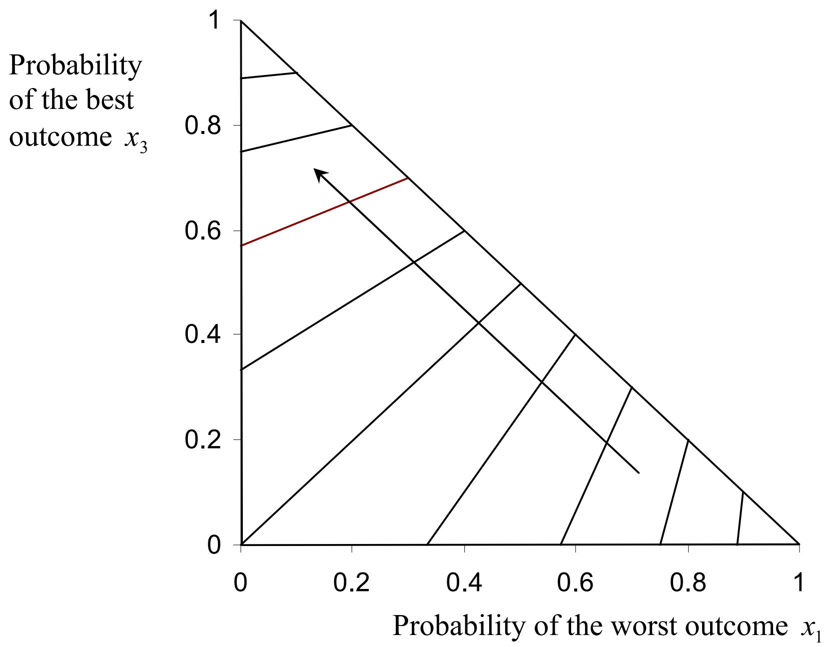

Figure 1 Family of indifference curves inside the probability triangle

3. Descriptive validity of proposed axioms

This section critically reviews the existing experimental evidence on the alleged violations of the four proposed axioms. To the best of my knowledge, there are no studies documenting any systematic violations of axiom 1 (solvability).

3.1. Violations of betweenness

Part b) of Axiom 2 (convexity) implies a betweenness axiom. Betweenness axiom states that if an individual is indifferent between two lotteries than a probability mixture of these two lotteries is equally good. A casual survey of the experimental literature from the late 1980's and early 1990's that tested betweenness suggests that it is not a descriptive axiom. However, when this empirical evidence is thoroughly examined, a more favorable picture emerges. The literature on stochastic utility (Loomes and Sugden 1998) reached a generic conclusion that some behavioral patterns, which appear as a systematic violation of a certain principle when taken at a face value, may actually support the principle once a stochastic specification is allowed. This generic conclusion applies to the case of betweenness.

3.1.1. Experimental evidence on betweenness

Coombs and Huang (1976) (experiment 1), Chew and Waller (1986), Camerer (1989), Battalio *et al.* (1990), Gigliotti and Sopher (1993) (experiments 1 and 3) and Camerer and Ho (1994) find that approximately 68% of subjects respect betweenness. The remaining subjects are split between quasi-convex (*i.e.* they dislike randomization) and quasi-concave (*i.e.* they like randomization) preferences approximately in a (non-corresponding) proportion of 24% to 8%. This alleged systematic violation of betweenness emerges when some lotteries used in the experiment are located on the edges of the probability triangle. Coombs and Huang (1976) (experiment 2), Camerer (1992), Starmer (1992) and Gigliotti and Sopher (1993) (experiment 2) find that approximately 76% of subjects respect betweenness and a split between quasi-convex and quasi-concave preferences is non-systematic (approximately in a non-corresponding proportion of 14% to 10%) when all of the lotteries used in the experiment are located inside the probability triangle.

Prelec (1990) finds that 76% of subjects reveal quasi-concave preferences and only 24% of subjects respect betweenness when the probability mass of the hypothetical lotteries is largely shifted to the worst outcome. Camerer and Ho (1994) replicate this result for one lottery triple "TUV" with real payoffs. Bernasconi (1994) finds a strong asymmetric violation of betweenness when betweenness is not a modal choice pattern in two lottery pairs (1 and 3).

3.1.2. A reexamination of experimental methodology

All studies mentioned in section 3.1.1 employ the same method to test betweenness. The subjects are asked to choose their most preferred lottery from three sets $\{L_1, L_2\}$, $\{L_1, M\}$ and $\{L_2, M\}$, where $M = \alpha L_1 + (1-\alpha)L_2$, $\alpha \in (0,1)$, and L_1, L_2 are arbitrary lotteries. Earlier studies typically consider only the first and the second pairwise choices. A particular fallacy of such a truncated experimental procedure is discussed below. If a probability mixture $\alpha L_1 + (1-\alpha)L_2$ is frequently (almost never) chosen in the second and third pairwise choices it has been interpreted as an evidence of quasi-concave (quasi-convex) preferences.

Consider a situation when the true preference of an individual is consistent with betweenness but it is distorted by an occasional random error. The distorting effect of an error is stronger when an individual chooses between the lotteries that have similar utility. This stochastic specification appears in Hey and Orme (1994), Camerer and Ho (1994) and Wu and Gonzalez (1996). For an alternative stochastic utility model, see Harless and Camerer (1994) p. 1261 who propose a constant choice-independent error rate.

A probability mixture *M* is located between lotteries L_1 and L_2 in terms of subjective utility, *i.e.* lotteries L_1 and L_2 are more distinct in terms of utility than lotteries *M* and L_1 or *M* and L_2. Therefore, the impact of a random error is more significant when an individual chooses between *M* and L_1 (L_2) than when an individual chooses between L_1 and L_2.

Alternatively, one can argue that the strength of an individual's preference relation is greater when an individual chooses between L_1 and L_2 (*e.g.* Loomes and Sugden, 1995).

Stochastic betweenness implies relation (2) where $*$ stands for $>, =, <, \geq$ or $\leq$.

$$\mathrm{Prob}(L_1 \succ L_2) * \frac{\mathrm{Prob}(M \succ L_2)}{\mathrm{Prob}(L_1 \succ M)} * 0.5 \qquad (2)$$

If an individual respects stochastic betweenness, a choice pattern when M is chosen either from the set $\{L_1, M\}$ or from the set $\{L_2, M\}$ (but not from both) should be a modal (most frequent) choice pattern. Additionally, if $\mathrm{Prob}(M \succ L_2) > \mathrm{Prob}(L_1 \succ M)$, M is more frequently chosen from the sets $\{L_1, M\}$ and $\{L_2, M\}$, which appears as if the evidence of quasi-concave preferences. If $\mathrm{Prob}(M \succ L_2) < \mathrm{Prob}(L_1 \succ M)$, M is less frequently chosen from the sets $\{L_1, M\}$ and $\{L_2, M\}$, which appears as if the evidence of quasi-convex preferences.

However, it is misleading to interpret these results as a systematic violation of betweenness. Both of these choice patterns are consistent with stochastic betweenness. A persuasive evidence of the systematic violations of betwenness would have been an asymmetric split between quasi-convex and quasi-concave preferences when betweenness is not a modal choice pattern. However, as described in section 3.1.1., such violations are rare.

If an asymmetric split between quasi-convex and quasi-concave preferences is caused by random errors, this split should be more symmetric when $\alpha = 0.5$, and more asymmetric when α is close to zero or one. Indeed, a highly asymmetric split between quasi-convex and quasi-concave preferences reported in Prelec (1990) and Camerer and Ho (1994) (triple TUV) is elicited for $\alpha = 1/17$, in Bernasconi (1994)—for $\alpha = 0.05$ and $\alpha = 0.95$.

Moreover, all empirical studies using $\alpha = 0.5$ except Camerer and Ho (1994) consider a binary choice only from two sets $\{L_1, L_2\}$ and $\{L_1, M\}$. An individual then reveals "as if" quasi-concave preference $M \succ L_1 \succ L_2$, if $\mathrm{Prob}(L_1 \succ L_2) > \mathrm{Prob}(L_1 \succ M)$. He or she reveals "as if" quasi-convex preference $L_2 \succ L_1 \succ M$, if $\text{Prob}(L_1 \succ L_2) < \text{Prob}(L_1 \succ M)$. Thus, a more frequent incidence of quasi-concave (quasi-convex) preferences is observed when $\text{Prob}(L_1 \succ L_2) > 0.5$ ($\text{Prob}(L_1 \succ L_2) < 0.5$). In the extreme case when $\text{Prob}(L_1 \succ L_2) = 0.99$ there may be still some chance to observe "as if" quasi-concave preferences $M \succ L_1 \succ L_2$ but almost no chance at all to observe "as if quasi-convex" preferences $L_2 \succ L_1 \succ M$.

3.2. Evidence for fanning-in

A survey of experiments testing the shape of individuals' indifference curves suggests that there is a non-negligible evidence for fanning-out going back to the Allais paradox (*e.g.* Allais, 1953) and common consequence and common ratio effects (Starmer, 2000). However, a universal fanning-out hypothesis (Machina, 1982) is rejected. There is a growing evidence that supports a universal fanning-in. This new evidence suggests that indifference curves tend to fan in when the probability mass is associated with the best and the worst outcome and tend to fan out when the probability mass is associated with medium outcomes. In addition, the evidence for fanning-in in all regions of the probability triangle has recently emerged.

Conlisk (1989) finds strong experimental support for the type of fanning-in implied by axiom 4 — 53% and 80% of subjects choose a more risky gamble in a common consequence problem when probability mass is largely shifted to the medium and the best outcome, correspondingly. This finding can be interpreted as an individual's indifference curves becoming almost horizontal when probability mass is largely shifted to the best outcome. Analogously, the so-called vertical fanning-in is documented in Starmer and Sugden (1989), Camerer (1989) p.92 and Battalio *et al* (1990). Wu and Gonzalez (1998) p.119 report a vertical fanning-in when the probability of the best outcome is above 0.33 and a vertical fanning-out when it is below 0.33.

Prelec (1990) and Kagel *et al.* (1990) find fanning-in when probability mass is largely shifted to the worst outcome. Wu and Gonzalez (1996) report the so-called horizontal fanning-in when a probability of the worst outcome is above 0.63 and a horizontal fanning-out when it is below 0.63. Camerer (1989) p.92 finds a similar evidence for small gains.

Bernasconi (1994) p.63 finds an experimental evidence for fanning-in by observing a reverse common ratio effect. Cubitt and Sugden (2001), Bosman and van Winden (2001) and Cubitt *et al* (2004) find an indirect evidence for a reverse common ratio effect in dynamic decision making under risk. Barron and Erev (2003) find a reverse common ratio effect in small feedback-based decision making. Battalio *et al* (1990) and Thaler and Johnson (1990) find an evidence for fanning-in *i.e.* an increased risk seeking for stochastically dominant lotteries when lotteries involve only guaranteed gains. Finally, Starmer (1992) and Humphrey and Verschoor (2004) find strong evidence consistent with a universal fanning-in in all regions of the probability triangle.

The above literature elicits fanning-in/out of an individual's indifference curves from an observed binary choice in a common consequence or common ratio problem involving lotteries typically defined on a common three-outcome structure. Therefore, the main findings from this literature can be summarized in the probability triangle presented in figure 2.

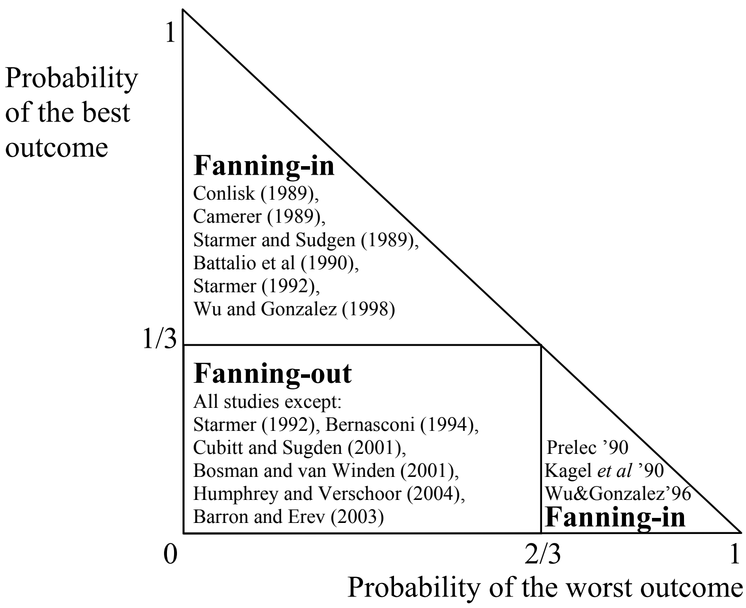

Figure 2 Empirical evidence for fanning-in

5. Conclusions

The proposed axiomatization explores theoretical features of an individual's preference for most probable winner in a binary choice under risk. Although such preference is implied by a simplistic behavioral rule (the heuristic of relative probability comparisons), I find some surprising perhaps even unexpected connections with other decision theories (skew-symmetric bilinear utility, weighted utility and regret theory). A preference for most probable winner is rationalized by four axioms: solvability, convexity, symmetry and a fanning-in. Notably, transitivity of preferences is not required. The present paper deals with binary choice; a natural extension of this work is to axiomatize a preference for most probable winner in a choice among many lotteries.

A preference for most probable winner falls into the betweenness class of decision theories that assume the linearity in probability of the sets $\{L : L \sim L_0\}, \forall L_0$. The alleged systematic violations of betweenness found in the experimental literature in the late 1980's and early 1990's can be explained within the concept of a stochastic utility developed in the mid 1990's. If the experimental evidence is reevaluated in the light of notions of stochastic utility, the betweenness axiom turns out to be quite descriptive.

Experimental evidence also emerges for universal fanning-in of indifference curves. However, this evidence seems to be stronger for some areas of the probability triangle than for others. The experimental evidence for the system of axioms proposed here to rationalize an individual's preference for most probable winner provides indirect evidence for the domain of applicability of the heuristic of relative probability comparisons.

A preference for most probable is a special case of a skew-symmetric bilinear utility theory and regret theory when outcomes are perceived as ordinal and the assumption of regret aversion is replaced with a preference for a win. Thus, an individual's preference for most probable winner is a simplified mirror image of regret theory.

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Appendix

Proof of theorem 2

According to theorem 1, axioms 1-3 hold if and only if there is a skew-symmetric function $\psi(.,.)$ such that for any two lotteries $L_1(p_1,...,p_n)$ and $L_2(q_1,...,q_n)$: $L_1 \succ L_2 \Leftrightarrow \sum_{i=1}^{n}\sum_{j=1}^{n} p_i q_j \psi(x_i, x_j) > 0$. For a specific pair of lotteries L_1 and L_2 described in the first part of axiom 4 the last statement can be rewritten as equation (3).

$$L_1 \sim L_2 \Leftrightarrow p_i p_j \psi(x_i, x_j) + p_i(1-p_j)\psi(x_i, x_n) + (1-p_i)p_j \psi(x_n, x_j) + (1-p_i)(1-p_j)\psi(x_n, x_n) = 0 \quad (3)$$

The right hand side of (3) can be rewritten as equation (4).

$$p_j = \frac{p_i \psi(x_i, x_n) + (1-p_i)\psi(x_n, x_n)}{p_i \psi(x_i, x_n) + (1-p_i)\psi(x_n, x_n) - p_i \psi(x_i, x_j) - (1-p_i)\psi(x_n, x_j)} \quad (4)$$

Taking the limit from both sides of (4) when $p_i \to 0$ we obtain that $\lim_{p_i \to 0} p_j = 0$ only if $\psi(x_n, x_n) = 0$. Furthermore, $\lim_{p_i \to 0} p_j / p_i = 1$ if and only if $\psi(x_i, x_n) = -\psi(x_n, x_j) = \psi(x_j, x_n)$ with the latter equality due to the skew-symmetric property of function $\psi(.,.)$. Following the same argument for a pair of lotteries L_3 and L_4 described in the second part of axiom 4 we obtain that $\psi(x_1, x_l) = \psi(x_1, x_k)$, $\forall k, l \in \{2,...,n\}$. Function $\psi(.,.)$ then has the following form:

$$\psi(x_i, x_j) = \begin{cases} a & i > j \\ 0 & i = j,\ a \neq 0, a = const \\ -a & i < j \end{cases} \quad (5)$$

Intuitively, the addition of axiom 4 imposes ordinality on the Fishburn's function $\psi(.,.)$. Thus, axioms 1-4 hold if and only if for any two lotteries $L_1(p_1,...,p_n)$ and $L_2(q_1,...,q_n)$: $L_1 \succ L_2 \Leftrightarrow \sum_{i=1}^{n}\sum_{j=1}^{n} p_i q_j \psi(x_i, x_j) > 0$, where function $\psi(.,.)$ is defined by equation (5). This last statement is algebraically equivalent to equation (1). *Q.E.D.*